B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

PRE BOARD-3, (2025-26)
MATHEMATICS (041) Set-2
Marking key

Class: XII | **Time: 3hrs**
Date: 06/01/26 | **Max Marks: 80**
Admission no: | **Roll no:**

General Instructions:

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1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment (04 marks each) with sub-parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks have been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	Section–A **This section comprises of MCQs of 1 mark each**	
Q.1	D	(1)
Q.2	C	(1)
Q.3	A	(1)
Q.4	D	(1)
Q.5	B	(1)
Q.6	D	(1)
Q.7	D	(1)
Q.8	A	(1)
Q.9	B	(1)
Q.10	C	(1)
Q.11	C	(1)
Q.12	A	(1)
Q.13	B	(1)
Q.14	**C**	(1)
Q.15	B	(1)
Q.16	C	(1)
Q.17	D	(1)
Q.18	A	(1)
	Followings are Assertion-Reason based questions in which a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true and R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true.	
Q.19	D	(1)
Q.20	A	(1)

	Section–B **This section comprises of very short answer type questions of 2 marks each**	
Q.21 Sol:	$\cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$ $\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$ OR $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3}$ $\frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$	(2)
Q.22 Sol:	$\frac{dy}{dx} = 2(\sin^{-1} x) \cdot \frac{d}{dx}(\sin^{-1} x) = \frac{2\sin^{-1} x}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\sqrt{1-x^2}\frac{dy}{dx}\right) = \frac{d}{dx}(2\sin^{-1} x)$ $\frac{-x}{\sqrt{1-x^2}}\frac{dy}{dx} + \sqrt{1-x^2}\frac{d^2y}{dx^2} = \frac{2}{\sqrt{1-x^2}}$ Multiplying the entire equation by $\sqrt{1-x^2}$ to eliminate the denominators yields: $-x\frac{dy}{dx} + (1-x^2)\frac{d^2y}{dx^2} = 2$	(2)
Q.23 Sol:	$\ln y = \ln((\tan x)^x) = x\ln(\tan x)$. $\frac{1}{y}\frac{dy}{dx} = (1)\cdot\ln(\tan x) + x\cdot\frac{d}{dx}(\ln(\tan x))$ $\frac{1}{y}\frac{dy}{dx} = \ln(\tan x) + x(\csc x \sec x)$ $\frac{dy}{dx} = (\tan x)^x(\ln(\tan x) + x\csc x\sec x)$	(2)
Q.24 Sol:	$\int_0^{2\pi} \lvert\sin x\rvert\, dx = \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$ $\int_0^{\pi} \sin x\, dx = [-\cos x]_0^{\pi} = (-\cos(\pi)) - (-\cos(0)) = (-(-1)) - (-1) = 1 + 1 = 2$	(2)

	$\int_{\pi}^{2\pi} (-\sin x)\, dx = [\cos x]_{\pi}^{2\pi} = (\cos(2\pi)) - (\cos(\pi)) = (1) - (-1) = 1 + 1 = 2$ $\int_{0}^{2\pi} \lvert \sin x \rvert\, dx$ is **4**. **(OR)** $A = \int_0^3 \sqrt{x}\, dx$ $A = \left[\frac{2}{3} x^{3/2}\right]_0^3 = \frac{2}{3}(3^{3/2}) - \frac{2}{3}(0^{3/2})$ $A = \frac{2}{3}(3\sqrt{3}) - 0 = 2\sqrt{3}$	
Q.25 SoL:	Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, its magnitude is: $\lvert\vec{a}\rvert = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ Given $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, its magnitude is: $\lvert\vec{a} \times \vec{b}\rvert = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$ We have $\lvert\vec{a} \times \vec{b}\rvert = \sqrt{2}$, $\vec{a} \cdot \vec{b} = 1$, and $\lvert\vec{a}\rvert = \sqrt{3}$. Let $\lvert\vec{b}\rvert = x$. $(\sqrt{2})^2 + (1)^2 = (\sqrt{3})^2 \cdot x^2$ $2 + 1 = 3x^2$ $3 = 3x^2$ $x^2 = 1$ x=1	(2)
	Section–C **This section comprises of short answer type questions of 3 marks each**	
Q.26 Sol:	Proper Graph The feasible vertices are $(40, 20)$, $(60, 30)$, $(120, 0)$, and $(60, 0)$. At **(40, 20)**: $Z = 5(40) + 10(20) = \mathbf{400}$ At **(60, 30)**: $Z = 5(60) + 10(30) = \mathbf{600}$ At **(120, 0)**: $Z = 5(120) + 10(0) = \mathbf{600}$. At **(60, 0)**: $Z = 5(60) + 10(0) = \mathbf{300}$. Max value is 600 at (120,0)	(3) 1 1 1
Q.27 Sol:	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at \sin t}{at \cos t} = \tan t$ $\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{at \cos t} = \frac{\sec^2 t}{at \cos t}$ $\frac{d^2y}{dx^2} = \frac{\sec^2 t}{at/\sec t} = \frac{\sec^3 t}{at}$ OR	(3) 1 1 1

Let $x = \sin\theta$ and $y = \sin\phi$. This means $\theta = \sin^{-1} x$ and $\phi = \sin^{-1} y$. The original equation becomes:

$$\sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$$

Substituting back $\theta = \sin^{-1} x$ and $\phi = \sin^{-1} y$:

$$\sin^{-1} x - \sin^{-1} y = \text{constant}$$

$$\frac{d}{dx}(\sin^{-1} x) - \frac{d}{dx}(\sin^{-1} y) = \frac{d}{dx}(\text{constant})$$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Q.28 Sol: (3)

$$f'(x) = \cos x - \sin x$$

$$\cos x - \sin x = 0 \implies \tan x = 1$$

Interval $[0, \frac{\pi}{4}]$: Test $x = \frac{\pi}{6}$. $f'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2} > 0$. The function is increasing.

Interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$: Test $x = \frac{\pi}{2}$. $f'(\frac{\pi}{2}) = 0 - 1 = -1 < 0$. The function is decreasing.

Interval $[\frac{5\pi}{4}, 2\pi]$: Test $x = \frac{3\pi}{2}$. $f'(\frac{3\pi}{2}) = 0 - (-1) = 1 > 0$. The function is Increasing.

Q.29 Sol: (3)

Let $u = \log x$. Then, the differential $du = \frac{1}{x}dx$ (or $x^{-1}dx$).

$$\int \frac{1}{u^2 - 5u + 4}\,du$$

$$\frac{1}{(u-1)(u-4)} = \frac{A}{u-1} + \frac{B}{u-4}$$

Solving for A and B, we find $A = -\frac{1}{3}$ and $B = \frac{1}{3}$.

$$\int\left(\frac{-\frac{1}{3}}{u-1} + \frac{\frac{1}{3}}{u-4}\right)du = -\frac{1}{3}\int\frac{1}{u-1}\,du + \frac{1}{3}\int\frac{1}{u-4}\,du$$

$$= -\frac{1}{3}\ln|u-1| + \frac{1}{3}\ln|u-4| + C = \frac{1}{3}\ln\left|\frac{u-4}{u-1}\right| + C$$

$$\frac{1}{3}\ln\left|\frac{\log x - 4}{\log x - 1}\right| + C$$

Q.30 Sol: **Proper Figure** (3)

$$A = \int_{-2}^{0} (x + 2 - 0)\, dx + \int_{0}^{2} (x + 2 - x^2)\, dx$$

$$\int_{-2}^{0} (x + 2)\, dx = \left[\frac{x^2}{2} + 2x \right]_{-2}^{0}$$

$$= \left(\frac{0^2}{2} + 2(0) \right) - \left(\frac{(-2)^2}{2} + 2(-2) \right)$$

$$= 0 - \left(\frac{4}{2} - 4 \right) = -(2 - 4) = 2 \text{ square units.}$$

$$\int_{0}^{2} (x + 2 - x^2)\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{0}^{2}$$

$$= \left(\frac{2^2}{2} + 2(2) - \frac{2^3}{3} \right) - \left(\frac{0^2}{2} + 2(0) - \frac{0^3}{3} \right)$$

$$= \left(2 + 4 - \frac{8}{3} \right) - 0$$

$$= 6 - \frac{8}{3} = \frac{18 - 8}{3} = \frac{10}{3} \text{ square units.}$$

$$A = 2 + \frac{10}{3} = \frac{6}{3} + \frac{10}{3} = \frac{16}{3} \text{ square units}$$

OR

$$\text{Area} = 4 \int_{0}^{4} \frac{3}{4} \sqrt{16 - x^2}\, dx = 3 \int_{0}^{4} \sqrt{16 - x^2}\, dx$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin\left(\frac{x}{a} \right). \text{ Here } a = 4:$$

$$\text{Area} = 3 \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \arcsin\left(\frac{x}{4} \right) \right]_{0}^{4}$$

$$\text{Area} = 3 \left[\left(\frac{4}{2} \sqrt{16 - 16} + 8 \arcsin\left(\frac{4}{4} \right) \right) - \left(\frac{0}{2} \sqrt{16 - 0} + 8 \arcsin\left(\frac{0}{4} \right) \right) \right]$$

$$\text{Area} = 3\left[(0 + 8 \arcsin(1)) - (0 + 0) \right] = 3 \left[8 \times \frac{\pi}{2} \right] = 12\pi$$

Q.31 Sol: (3)

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\vec{a}_2 - \vec{a}_1 = (3\hat{i} + 5\hat{j} + 7\hat{k}) - (-\hat{i} - \hat{j} - \hat{k}) = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = |\hat{i} \quad \hat{j} \quad \hat{k}\ 2 \quad -6 \quad 1\ 1 \quad -2 \quad -2| = \hat{i}((-6)(-2) - (1)(-2)) - \hat{j}((2)(-2) - (1)(1))$$

$$= \hat{i}(12 + 2) - \hat{j}(-4 - 1) + \hat{k}(-4 + 6) = 14\hat{i} + 5\hat{j} + 2\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{14^2 + 5^2 + 2^2} = \sqrt{196 + 25 + 4} = \sqrt{225} = 15$$

$$d = \frac{|102|}{15} = \frac{102}{15} = \frac{34}{5} = 6.8 \text{ units}$$

OR

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 : \ \vec{b} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

	The equation of a line in vector form is $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar parameter. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$	
	Section–D **This section comprises of long answer type questions of 5 marks each**	
Q.32 Sol:	$\lvert A\rvert$=-1 $A^{-1} = \begin{vmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{vmatrix}$ X= A^{-1}B X= 1, y=2, Z=	(5)
Q.33	$\frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$ $v + x\frac{dv}{dx} = v - \sin^2(v)$ $x\frac{dv}{dx} = -\sin^2(v)$ $\frac{dv}{\sin^2(v)} = -\frac{dx}{x}$ $\int \csc^2(v)\,dv = -\int \frac{1}{x}\,dx$ $-\cot(v) = -\ln\lvert x\rvert + C$ $\cot\left(\frac{y}{x}\right) = \ln\lvert x\rvert - C$ Use the given condition $y = \frac{\pi}{4}$ when $x = 1$ to find the constant C: $\cot\left(\frac{\pi/4}{1}\right) = \ln\lvert 1\rvert - C$ $\cot\left(\frac{\pi}{4}\right) = 0 - C$ $1 = -C$ $C = -1$ $\cot(\frac{y}{x}) = \ln\lvert x\rvert + 1.$ OR	(5)

	$I(x) = e^{\int -3\cot(x)dx} = e^{-3\ln\lvert\sin(x)\rvert} = e^{\ln(\lvert\sin(x)\rvert^{-3})} = \csc^3(x)$ $$\frac{d}{dx}\left(y\csc^3(x)\right) = \sin(2x)\csc^3(x)$$ Simplify the right side using $\sin(2x) = 2\sin(x)\cos(x)$ and $\csc(x) = \frac{1}{\sin(x)}$. $$\sin(2x)\csc^3(x) = 2\sin(x)\cos(x)\frac{1}{\sin^3(x)} = \frac{2\cos(x)}{\sin^2(x)} = 2\cot(x)\csc(x)$$ Integrate both sides with respect to x: $$\int \frac{d}{dx}\left(y\csc^3(x)\right)dx = \int 2\cot(x)\csc(x)dx$$ $$y\csc^3(x) = -2\csc(x) + C$$ Use the given condition $y = 2$ when $x = \frac{\pi}{2}$ to find the value of the constant C. $$2 = -2\sin^2\left(\frac{\pi}{2}\right) + C\sin^3\left(\frac{\pi}{2}\right)$$ $$2 = -2(1)^2 + C(1)^3$$ $$2 = -2 + C$$ $$C = 4$$ $\mathbf{y = 4\sin^3(x) - 2\sin^2(x)}$	
Q.34	$$\frac{x\tan x}{\sec x + \tan x}\cdot\frac{\sec x - \tan x}{\sec x - \tan x} = x\frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x}$$ Since $\sec^2 x - \tan^2 x = 1$, the expression simplifies to $x(\sec x\tan x - \tan^2 x)$. Using the identity $\tan^2 x = \sec^2 x - 1$, the integral becomes: $$I = \int x(\sec x\tan x - (\sec^2 x - 1))\,dx = \int x(\sec x\tan x - \sec^2 x + 1)\,dx$$ $$I = \int x\sec x\tan x\,dx - \int x\sec^2 x\,dx + \int x\,dx$$	(5)

	Apply integration by parts $\int u\,dv = uv - \int v\,du$ to the first two integrals. For the first integral, let $u = x$, $dv = \sec x \tan x\,dx$. Then $du = dx$, $v = \sec x$. $$\int x \sec x \tan x\,dx = x\sec x - \int \sec x\,dx$$ For the second integral, let $u = x$, $dv = \sec^2 x\,dx$. Then $du = dx$, $v = \tan x$. $$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$$ $$I = (x\sec x - \int \sec x\,dx) - (x\tan x - \int \tan x\,dx) + \int x\,dx$$ $$I = x\sec x - x\tan x - \int \sec x\,dx + \int \tan x\,dx + \int x\,dx$$ $$I = x(\sec x - \tan x) - \ln\|\sec x + \tan x\| + \ln\|\sec x\| + \frac{x^2}{2} + C$$ **OR**	
Q.35	$\frac{x-2}{1} = \frac{y-0}{2} = \frac{z-2}{-3} = \lambda$ $M = (2+\lambda, 2\lambda, 2-3\lambda)$. $\vec{PM}\cdot\vec{d} = (3+\lambda)(1) + (2\lambda-5)(2) + (-3\lambda)(-3) = 0$ $$3 + \lambda + 4\lambda - 10 + 9\lambda = 0$$ $$14\lambda - 7 = 0 \Longrightarrow \lambda = 0.5$$ $$(2.5, 1, 0.5) = \left(\frac{-1+x'}{2}, \frac{5+y'}{2}, \frac{2+z'}{2}\right)$$ Solving for the coordinates yields $P' = (6, -3, -1)$. $$d = \sqrt{(6-(-1))^2 + (-3-5)^2 + (-1-2)^2}$$ $$d = \sqrt{7^2 + (-8)^2 + (-3)^2} = \sqrt{49+64+9} = \sqrt{122}$$	(5)
	Section–E **This section comprises of case based questions**	
Q.36	(i) $\mathbf{3x + 2y = 12}$ (ii) $A(x) = x(6 - \frac{3}{2}x) + \frac{\sqrt{3}}{4}x^2$, (iii) $x = \frac{12}{6-\sqrt{3}}$, $y = \frac{18 - 6\sqrt{3}}{6-\sqrt{3}}$ m.	(2) (2)

	Or $P(x) = (3 - \frac{\sqrt{3}}{2})x + \frac{100}{x}$.	
Q.37	(i) $\cos(\theta) = \frac{\vec{DV} \cdot \vec{DA}}{\lvert\vec{DV}\rvert \lvert\vec{DA}\rvert}$, $\cos(\theta) = \frac{11}{\sqrt{90}\sqrt{45}} = \frac{11}{\sqrt{4050}} \approx 0.1736$ (ii) $\text{Proj}_{\vec{DA}} \vec{DV} = \frac{11}{\sqrt{45}}$	(1) (1) (2)
Q.38	(i) The relations which are functions from A to B are $\boldsymbol{R_3}$**,** $\boldsymbol{R_4}$**, and** $\boldsymbol{R_5}$. (ii) The only relation which is injective is $\boldsymbol{R_4}$. (iii) (A) The function $g(x) = x^2$ is **injective but not surjective**. (B) The relation R in roster form is $\mathbf{R} = \{(\mathbf{1},\mathbf{3}),(\mathbf{2},\mathbf{6}),(\mathbf{3},\mathbf{9}),(\mathbf{4},\mathbf{12}),(\mathbf{5},\mathbf{15}),(\mathbf{6},\mathbf{18}),(\mathbf{7},\mathbf{21}),(\mathbf{8},\mathbf{24}),(\mathbf{9},\mathbf{27}),(\mathbf{10},\mathbf{30})\}$. The relation is **not reflexive, symmetric, or transitive.**	(2) (2)

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